1. Given relational schema $R(ABCDE)$ with following functional dependencies:

\[ AB \rightarrow CDE, B \rightarrow C, C \rightarrow D, E \rightarrow A \]

Determine all candidate keys and create the BCNF step by step.

2. Given following relational schema

\[ car\_sale (plate\_number, seller, sale\_date, commision, discount) \]

with following functional dependencies:

- sale_date $\rightarrow$ discount
- seller $\rightarrow$ commision

To which normal form does this relational schema apply? If necessary, transform the schema into 3. normal form. Is the schema minimal?

3. Given following set of functional dependencies $\Sigma = \{ A \rightarrow B; BC \rightarrow A \}$ on the relational schema $R(ABCD)$. Specify at least one relation $r$ over the schema $R$ that contradicts all functional dependencies. Explain your answer. (Assume that all attributes are of type integer.)

4. Design a database schema according to the given functional dependencies

\[ A \rightarrow B, AB \rightarrow C, A \rightarrow C, B \rightarrow A, C \rightarrow D, D \rightarrow E \]

(a) Using the decomposition approach.

(b) Using the synthesis approach.
5. A database designer decomposed relation $R(ABCDE)$ into relations $R_1(ABC)$ and $R_2(CDE)$. State at least two functional dependencies so that the decomposition is:

(a) neither lossless nor dependency preserving
(b) lossless but not dependency preserving
(c) not lossless but dependency preserving
(d) lossless as well as dependency preserving

Good Luck!