Motivation

- Fact tables in data warehouses can become very large, such that a full table scan becomes un advantageous.
- Example: Scan over 10 GB table at 10 MB/s = ca. 17 minutes.
- Queries just affect a relatively small part of the available data:
  - Depending on the restrictions on individual dimensions the result set of a query may affect a few percent or per mille (or even less) of the data.
- Use of index structures to minimize the number of required page accessess.
Classification of index structures

- **Clustering**: data that are likely to often processed together will also be stored physically close to each other
  - *Tuple Clustering*: Storing of tuples on the same physical page
  - *Page Clustering*: storing related pages close together in secondary storage (allows *prefetching*)

- **Dimensionality**: specify how many attributes (dimensions) of the underlying relation for calculation of the index key can be used

- **Symmetry**: If the performance is independent of the order of the index attributes, we have a symmetrical index structure, otherwise asymmetrical

- **Tuple references**: Type of tuple references within the index structure

- **Dynamic behavior**: Effort to update the index structure for insert, update and delete; (and possibly problem of "degeneration")
Comparison of Index Structures

- Full Table Scan
- Clusternder Primärindex
- Mehrere Sekundärindexe, Bitmap-Indexte
- Multidimensionaler Index
One dimensional tree structures

- B-Baum [Bayer/McCreight 1972]

```
40 43 52
59 62
70 74 75
```

```
82 84 87
89 90 97
```
B-Tree

Order of a B Tree: min. number of entries on the index pages (except for the root page)

Definition: Index tree is a B Tree of order \( m \), if

- Each page contains at most \( 2m \) elements
- Each page except for the root page contains at least \( m \) elements
- Each page is either a leaf side without successors or it has \( i + 1 \) successors (\( i \): number of elements)
- All leaf pages are on the same level
B-Tree: Properties

- $n$ records in the main file
  - $\log_m(n)$ page accesses from the root to the leaf
- Balance criterion leads to almost complete evenness
- Insert, delete, search with $O(\log_m(n))$
- Memory space utilization: at least mindestens 50% (except for root)
$B^+$-Tree

- $B^+$ Tree (variant of the $B$-Tree): Tupels/TIDs only in the leaves; leaved among themselves chained for a sequential (range) pass.
Properties of $B$- and $B^+$-Trees

- One-dimensional structure (index on an attribute)
- As a primary index tuples can be stored directly in the tree (allowing simple clustering, especially in the $B^+$-Tree)
- As a secondary index only TIDs are stored in the tree
- Balanced Trees (Path from root to leaf has everywhere the same length); balancing requires more effort in reorganization in case of updates on the data
  - For data warehouses is of minor importance
- $B^+$-tree especially suited for range queries (by concatenation on leaf-level)
Use of $B^−/B^+-$Trees

- $B^-$ and $B^+-$Trees: one-dimensional structures:
  - Only insufficient support of multidimensional queries

- Possible applications with multi-dimensional queries
  - For each attribute involved, there is a $B^-$ or $B^+-$Tree as a secondary index
  - Then for each attribute the set of TIDs is determined depending on whether they fulfill the query restriction
  - Now the intersection of the independently from each other determined TID sets is taken, the corresponding tuples form the query result
B⁺-Tree: Conclusion

- Robust, generic data structure
- Independent of data type (only order required)
- Efficient update algorithms
- Compact
- "Working Horse" of all RDBMS

Problems
  - Attributes of low cardinality → degenerated trees
  - Composed Indexes → order sensitive
Degenerated B-Trees

- Example:
  - Table Customer
  - Attributes: among others Gender (m, f)
  - Index

```sql
CREATE INDEX s_idx ON Customer(Gender)
```

![Diagram showing a degenerated B-tree with keys 0x100, 0x101, 0x105, 0x110, ... and 0x102, 0x103, 0x104, 0x106, ...]
Order Dependency

- Composed Index
  - Indexing of concatenated attribute values
  - Problem: Adjust the order of the query predicates

Example:
- Table: Customer
- Attributes: cclass, gender, profession
- Index:
  ```
  CREATE INDEX csp_idx
  ON Customer(cclass, gender, profession)
  ```
- Queries:
  ```
  SELECT ... FROM ...
  WHERE cclass=1 AND
  gender='m' AND profession='Lecturer'
  ```
B+-Tree-Tricks: Oversized Index

Query:

```
SELECT AVG(Age)
FROM Customer
WHERE cclass=1 AND gender='m'
AND profession='Lecturer'
```

Index usage
- Searching the value "1|m|Lecturer"
- Access to a block of the relation Customer over TID for the value of Age

Better:

```
CREATE INDEX csp_idx
ON Customer(cclass, gender, profession, age)
```
B⁺-Tree-Tricks: Calculated Indexes

- Calculation of indexed values by using a function
- Example: Index over `Customer(name)`
  - Query:
    ```sql
    SELECT * FROM Customer
    WHERE name="Müller" OR name="mueller"
    OR name="Mueller" ...
    ```
  - Index usage not possible
- Better
  ```sql
  CREATE INDEX n_idx ON Customer(upper(name))
  CREATE INDEX n_idx ON Customer(soundex(name))
  ```
Oracle9i: Special Features

- **Index Skip Scan**: uses composed indexes also when the is not in the condition

**Example:**
- Table `Customer`, Index on `(status, registration#)`
- Index usable for a query with

```plaintext
... WHERE registration# = 4245
```

- Searching the secondary index for all `DISTINCT` values of the first attribute
- Only useful if the first attribute has a low cardinality
Oracle9i: Special Features (2)

- Index-organized tables
  - Tuples are additionally stored in a $B^+$-Tree
  - No indirection via TID necessary

- User-defined indexes
  - Implementation of own index structures for user-defined data types
  - Transparent use
  - Specify own cost estimates
Bitmap Indexes

- Idea: **Bit-Array** to encode the tuple attribute value mapping
- Comparison with tree-based index structures:
  - Avoids degenerated B-trees
  - Insensitive towards higher number of dimensions
  - Easier support of queries, where only some (the indexed) dimensions are restricted
  - But for generally higher update costs
  - In data warehouses mostly unproblematic due to the majority of read-only accesses unproblematisch
### Bitmap Index: Implementation

- **Principle:** Replacement of TIDs (rowid) for a key value in the $B^+$-Tree by bit list
- **Node structure:**

  
  | B: 010010...01 | F: 101000...10 | O: 000101...00 |

- **Advantage:** lower storage consumption
  - Example: 150,000 Tuples, 3 different key values, 4 Bytes for a TID
    - $B^+$-Tree: 600 KB
    - Bitmap: $3 \cdot 18750 \text{ Byte} = 56$ KB

- **Disadvantage:** higher update effort
Bitmap Index: Implementation (2)

- Definition in Oracle
  
  ```sql
  CREATE BITMAP INDEX orderstatus_idx
  ON Order(status);
  ```

- Use particularly for Star-Query transformation (Join between dimension- and fact table)
- Storage in compressed form
- Furthermore: Bitmap-based join indexes
Standard Bitmap Index

- Each Dimension is stored separately
- For each attribute value, an individual bitmap vector is created:
  - For each tuple, there is a corresponding bit that is set to 1 when the indexed attribute in the tuple contains the reference value of this bitmap vector
  - The number of resulting bitmap vectors per dimension corresponds to the number of different values that occur for the attribute
Standard Bitmap Index (2)

- Example: Attribute Gender
  - 2 Feature values (m/f)
  - 2 Bitmap vectors

<table>
<thead>
<tr>
<th>PersId</th>
<th>Name</th>
<th>Gender</th>
<th>Bitmap-f</th>
<th>Bitmap-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>007</td>
<td>James Bond</td>
<td>M</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>008</td>
<td>Amelie Lux</td>
<td>F</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>010</td>
<td>Harald Schmidt</td>
<td>M</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>011</td>
<td>Heike Drechsler</td>
<td>F</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

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Standard Bitmap Index (3)

- Selection of tuples can be achieved by linking bitmap vectors.
- Example: Bitmap index over the attributes gender and month of birth:
  - 2 Bitmap vectors B-f and B-m for gender.
  - 12 Bitmap vectors B-1, ..., B-12 for the months, if all months occur.
- Query: "all women born in march"
  - Calculation: B-f \(\land\) B-3 (bitwise conjunctively linked).
  - Result: all tuples, at whose position in the bitmap vectors a 1 is given in the result.
Multicomponent Bitmap Index

- For the Standard Bitmap Indexes many bitmap vectors are created for attributes with many feature values.
- \(< n, m >\) Multicomponent Bitmap Index allow to index \(n \cdot m\) possible feature values by using \(n + m\) bitmap vectors.
- Each value \(x(0 \leq x \leq n \cdot m - 1)\) can be represented by \(y\) and \(z\):

\[ x = n \cdot y + z \text{ mit } 0 \leq y \leq m - 1 \text{ und } 0 \leq z \leq n - 1 \]

- At maximum \(m\) bitmap vectors for \(y\) and \(n\) bitmap vectors for \(z\).
- Storage overhead is reduced from \(n \cdot m\) to \(n + m\) vectors.
- However, point queries require reads of 2 bitmap vectors.
Multicomponent Bitmap Index (2)

- Example: Two Component Bitmap Index
- For $M = 0..11$ for instance $x = 4 \cdot y + z$
- $y$ values: B-2-1, B-1-1, B-0-1
- $z$ values: B-3-0, B-2-0, B-1-0, B-0-0

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>B-2-1</td>
<td>B-1-1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Example: ZIP-Codes

- Encoding of ZIP-Codes
- Values from 00000 to 99999
- Direct Implementation: 100.000 columns
- Two Component Bitmap Index (first 2 digits + 3 last digits): 1.100 columns
- Five Components: 50 columns
  - Suitable for range queries "ZIP 39***"
- Binary encoded (to $2^{17}$): 34 columns
  - Only for point queries!

Note: Encoding to the base 3 results in 33 columns....
Range Encoding Bitmap Index

- Standard and Multicomponent Bitmap Indexes
  - Well suited for point queries
  - Inefficient for large ranges, because many bitmap vectors need to be linked

- Idea of range encoded bitmap indexes:
  *In the bitmap vector a bit is set to 1 if the value of the attribute is smaller or equal to the given value.*

- Range query $2 \leq attr \leq 7$ requires only 2 bitmap vectors: B-1 and B-7.

- Resulting bitmap vector is $((\neg B-1) \land B-7)$.
  - For range queries at maximum 2 bitmap vectors need to be read (only one for one-side restricted ranges)
  - For point queries exactly 2 bitmap vectors need to be read
Range Encoding Bitmap Index

<table>
<thead>
<tr>
<th>Month</th>
<th>Dec</th>
<th>Nov</th>
<th>Oct</th>
<th>Sep</th>
<th>Aug</th>
<th>Jul</th>
<th>Jun</th>
<th>May</th>
<th>Apr</th>
<th>Mar</th>
<th>Feb</th>
<th>Jan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B-11</td>
<td>B-10</td>
<td>B-9</td>
<td>B-8</td>
<td>B-7</td>
<td>B-6</td>
<td>B-5</td>
<td>B-4</td>
<td>B-3</td>
<td>B-2</td>
<td>B-1</td>
<td>B-0</td>
</tr>
<tr>
<td>Junq - 5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>April - 3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Jan. - 0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Feb. - 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>April - 3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dec. - 11</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Aug. - 7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sept. - 8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Range query $February \leq Date \leq August$ requires B-0 and B-7.
- Resulting bitmap vector is $((\neg B-0) \land B-7)$
Multicomponent Range Encoding Bitmap Index

- Combination of both techniques
- First, a multicomponent bitmap index is created
- On each built group of bitmap vectors the range encoding is applied
  - Due to the multicomponent technique less storage consumption (because of the smaller number of bitmap vectors)
  - The range encoding allows for efficient support of range queries
  - Because of the range encoding there is always one bitmap vector (component) dispensable in each group (representing the value $n - 1$ and $m - 1$, respectively, since there always all bits have to be set to 1 da dort immer alle Bits auf 1); hence only $n + m - 2$ bitmap vectors are needed
Example MCREBMI

**Example Multicomponent Range Encoding Bitmap Index**

- $B-0-1' = B-0-1$
- $B-1-1' = B-1-1 \lor B-0-1'$
- $B-2-1' = B-2-1 \lor B-1-1' = B-2-1 \lor B-1-1 \lor B-0-1 = 1$

<table>
<thead>
<tr>
<th>M</th>
<th>B-1-1'</th>
<th>B-0-1'</th>
<th>B-2-0'</th>
<th>B-1-0'</th>
<th>B-0-0'</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Interval Encoding Indexing

- Principle: each Bitmap vector represents a defined interval

<table>
<thead>
<tr>
<th>M</th>
<th>I-5</th>
<th>I-4</th>
<th>I-3</th>
<th>I-2</th>
<th>I-1</th>
<th>I-0</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Query: $(2 \leq M \leq 8) \iff$ Evaluation of $I_2 \lor I_3$
Selection of Index Structures

- In Data Warehouses normally multidimensional range queries
- Selection of index structure dependent on query profile
  - Is a specific attribute predominantly restricted?
    ⇒ For asymmetrical index structures, the order of the index attributes shall be selected according to their frequency in the query profile
  - When no attribute can be identified as particularly important or many ad-hoc queries occur, symmetrical structures (secondary indexes, multidimensional indexes) are advantageous
Selection of Index Structures (2)

- **Standard Bitmap Index**
  - Fast, efficient implementation
  - Much storage space needed for a large number of feature values

- **Multicomponent Bitmap Index**
  - For point queries smallest number of read operations

- **Range Encoding Bitmap Index**
  - One-side restricted range queries

- **Interval Encoding Bitmap Index**
  - Two-side restricted range queries
Join Index

- Accelerating join computations by indexing attributes of "foreign" relations
- Precomputation of the join and storing as an index structure
- Join/Grouping partially without access to foreign relation (e.g., dimension table) possible
### Join Index: Example

<table>
<thead>
<tr>
<th>V_ROWID</th>
<th>GeoID</th>
<th>TimeID</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x001</td>
<td>101</td>
<td>11</td>
<td>200</td>
</tr>
<tr>
<td>0x002</td>
<td>101</td>
<td>11</td>
<td>210</td>
</tr>
<tr>
<td>0x003</td>
<td>102</td>
<td>11</td>
<td>190</td>
</tr>
<tr>
<td>0x004</td>
<td>102</td>
<td>11</td>
<td>195</td>
</tr>
<tr>
<td>0x005</td>
<td>103</td>
<td>11</td>
<td>100</td>
</tr>
<tr>
<td>0x006</td>
<td>103</td>
<td>11</td>
<td>95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>G_ROWID</th>
<th>GeoID</th>
<th>Branch</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x100</td>
<td>101</td>
<td>Allee-Center</td>
<td>Magdeburg</td>
</tr>
<tr>
<td>0x101</td>
<td>102</td>
<td>Bördepark</td>
<td>Magdeburg</td>
</tr>
<tr>
<td>0x102</td>
<td>103</td>
<td>Anger</td>
<td>Erfurt</td>
</tr>
<tr>
<td>0x103</td>
<td>104</td>
<td>Erfurter Str.</td>
<td>Ilmenau</td>
</tr>
</tbody>
</table>

**CREATE INDEX** `joinidx` **ON** `Sales(Geography.GeoID)`

**USING** `Sales.GeoID = Geography.GeoID`

<table>
<thead>
<tr>
<th>0x100: { 0x001, 0x002, ...}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x101: { 0x003, 0x004, ...}</td>
</tr>
<tr>
<td>0x103: { ...}</td>
</tr>
</tbody>
</table>
Bitmap Join Index

- So far:
  - Predicates for bitmap indexes not applied on foreign keys
  - Join has to be still executed

- Bitmap indexes only for Star Join optimization helpful

- Combination of
  - Bitmap Index and
  - Join Index
Bitmap Join Index with Oracle

**Definition**

```sql
CREATE BITMAP INDEX join_idx
ON Sales(Geography.GeoID)
FROM Sales, Geography
WHERE Sales.GeoID = Geography.GeoID
```

- Makes joins redundant (no access to region needed)
- Linking with other bitmap indexes on table sales possible

```sql
SELECT SUM(Sales.sales)
FROM Sales, Geography
WHERE Sales.GeoID = Geography.GeoID AND Geography.Stadt = 'Magdeburg'
```

**Infos:** Oracle Dokumentation 11g2 - Part E25789-01
Indexed Views

- SQL Server 2008: Indexing Views
- Materialization of affected data
- Automated update for changes in the base data → materialized views

```sql
CREATE VIEW Sales2009 AS
SELECT City, Sales, S.TimeID, S.GeographyID
FROM Sales S, Time T, Geography G
WHERE S.TimeID = T.TimeID AND T.Year = 2009
    AND S.GeographyID = G.GeographyID;

CREATE UNIQUE CLUSTERED INDEX V2009_IDX
    ON Sales2009(TimeID, GeographyID);
```
Multidimensional Index Structures

- **Hash-based Structures**
  - Grid Files
  - Multidimensional Dynamic Hashing

- **Tree-based Structures**
  - kdB-Tree
  - R-Tree [Gutman 1984]
  - UB-Tree [Bayer 1996]
Grid File

- Multidimensional form of data organization
  - Combination of key transformation elements (Hash approaches) and Index Files (Tree approaches)

- Idea
  - Dimension refinement: Equal distribution of the multidimensional space of the chosen dimension through a complete cut (insertion of a hyperplane)
Grid File: Principles

- Separation of the data space in squares (search region: \(k\)-dimensional cuboids)
- Neighborhood preservation: Storage of similar objects on the same page
- Symmetrical treatment of all space dimensions: partial match queries
- Dynamic adjustment of the structure during inserts and deletes
- **Principle of 2 disc accesses** for exact match queries
Grid File: Structure

- **Grid**: $k$ one dimensional fields (Scales), each scale represents an attribute
- **Scales**: consists of partitions from the mapped value space in intervals
- **Grid Directory**: consists of Grid cells, which dissect the data space in squares
- **Grid Cells**: form a Grid region, that is assigned to exactly one record page
- **Grid Region**: $k$-dimensional, convex construction (pairwise disjoint)
Grid File: Structure (2)
Grid File: Operations

- Beginning state: Cell = Region = one record page
- Page overflow
  - Dividing of pages
  - If there is just one cell in the region belonging to a page: Segmentation of the interval on a scale
  - If region consists of multiple cells: Division of those cells in separate regions
- Page underflow
  - Subsumption of two regions if the result is a convex region
Multidimensional Hashing – MDH

- Idea: Bit Interleaving
- Calculate in diverging order the address bits of the access attributes
- Example: two dimensions

<table>
<thead>
<tr>
<th></th>
<th><em>0</em>0</th>
<th><em>0</em>1</th>
<th><em>1</em>0</th>
<th><em>1</em>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0<em>0</em></td>
<td>0000</td>
<td>0001</td>
<td>0100</td>
<td>0101</td>
</tr>
<tr>
<td>0<em>1</em></td>
<td>0010</td>
<td>0011</td>
<td>0110</td>
<td>0111</td>
</tr>
<tr>
<td>1<em>0</em></td>
<td>1000</td>
<td>1001</td>
<td>1100</td>
<td>1101</td>
</tr>
<tr>
<td>1<em>1</em></td>
<td>1010</td>
<td>1011</td>
<td>1110</td>
<td>1111</td>
</tr>
</tbody>
</table>
Idea MDH

- MDH builds upon *linear Hashing*
  - Dynamic Hash Method
  - Bit sequence prefixes address storage blocks
- Hash values are bit sequences, each having a beginning section serving as a current hash value
  - For binary numbers: often inverting the bit representation before prefix computation
- Compute one bit string per involved attribute
- Traverse beginning sections according to the principle of bit interleaving in a cyclic manner
- Hash values are composed of the surrounding bits of the individual values
- Family of Hash functions $h_i$ for bit sequences of length $i$
  - Dynamic growth: go from $i$ to $i + 1$
MDH Illustration

- Clarifies composition of the hash function $h_i$ for three dimensions and the value $i = 7$
- At $i = 8$ a further bit of $x_2$ is used (more specific: of $h_{8_2}(x_2)$)
- MDH Complexity
  - Exact Match Queries: $O(1)$
  - Partial Match Queries, $t$ of $k$ attributes set, Effort $O(n^{1-\frac{t}{k}})$
  - Follows from the number of pages when certain bits are "unknown"
  - Special cases: $O(1)$ for $t = k$ and $O(n)$ for $t = 0$
**kB-Tree**

- **k-dimensional index trees**
  - kd-Tree: binary Tree for multidimensional basic structure; Main memory storage algorithms [Bentley 1975]
  - kdB-Tree: Combination of kd-Tree and B-Tree (higher branching degree)
  - kdB-Tree: Improving the kd-Tree

- **Idea of the kdB-Tree**
  - On each index page a subtree is presented, which branches after multiple subsequent attributes
  - Effort: exact-match \(O(\log n)\), partial match better than \(O(n)\)
kdB-Tree: Structure

- kdb-Tree of type \((b, t)\)
- Range pages (inner nodes): contain kd-Tree with max. \(b\) inner nodes
  - kd-Tree with with split elements and two pointers
  - Split element: Access attribute and value
  - Left pointer: smaller access attributes
  - Right pointer: larger access attributes
- Record pages (leaves): contain up to \(t\) tuples of the stored relation
kdB-Tree: Example

Multidimensional Index Structures
kdB-Tree: Split attributes

- **Order**
  - Cyclic
  - Consideration of selectivity: Access attribute with high selectivity ideally early and used more often than split element

- **Split attribute value**
  - Finding a suitable average of the value space given distribution information
k[dD](B)-Tree: Conclusion

- Stores also bad distributed data
- Difficult to handle for more than three dimensions
R-Tree

- Each node of the R-Tree stores max. $m$ index entries ($m = 2d + 1$)
- Each node (except for the root) contains at least $n$ entries ($n = d + 1$)
- $d$-dimensional R-Tree uses $d$-dimensional intervals (rectangles, squares) for indexing the data space
- Entry on leaf level: $(I, tid)$ with $I$ a $d$-dimensional interval and $tid$ tuple identifier, that references the respective tuple
- Entry in inner nodes: $(I, cp)$ with $I$ the $d$-dimensional interval, that contains all intervals of the child node entries (minimum bounding box) and $cp$ pointers to this child node (child pointer)
R-Tree (2)

Special Features (in comparison to the $B^*$-Tree)

- Search: if different regions of the nodes on the same level overlap, multiple descendants may need to be traversed even for point queries.
- Insert: attempt of finding an interval that does not need to be extended, otherwise the interval that has to be extended the least.
- The deletion of data usually does not play a role in Data Warehouses; inserts only in big time intervals (but in turn often with many new tuples).
  - Efficient method important to built the R-Tree structure in a bottom-up manner.
R-Tree: Example
R-Tree: Conclusion

- Better adjustment of the regions to the data
R⁺-Tree

- Basic idea: **Forbid overlaps**
  - Adjustment of multiple nodes
  - Adjustment implies dissections in smaller MBRs without previous overflow → nodes with few tuples (unused capacity) → many nodes (Degeneration)
  - Clipping for storing geometric objects
R⁺-Baum: Example
R*-Tree

- Minimizing overlaps
- However, not forbidden!
R*-Tree

- Support for OLAP
- Predominant for: Aggregation functions
  → SUM, MAX, AVG, COUNT, ...
- R*-Tree stores selected aggregated values of lower nodes in each inner node (materialized views)
  → a stands for aggregated
R*-Tree II

R*-Tree for COUNT and SUM
UB-Tree

- Data space is dissected in disjoint subspaces using a space-filling curve (often the so-called Z-Curve)
- Each point of the attributes to be indexed within the multidimensional space is projected to a scalar value. the Z-Value
- Z-Values are used as keys in a standard $B^+$-Tree
UB-Tree (2)

- Disection of a 2-dimensional space with the Z-Curve:
UB-Tree (3)

- **Z-Values** can be computed efficiently (in linearer time):
  - Per dimension the basic intervals are binary numbered;
  - By interleaving the bits, the respective Z-Value is obtained.

- **Z-Region**: is determined by an interval $[a, b]$ of Z-Values
  - Z-Regions of a UB-Tree are adjusted dynamically so that the objects within a Z-Region fit exactly within a page of a $B^+$-Tree
  - With that a $B^+$-Tree can be used as a basic structure
UB-Tree (4)
UB-Tree: Region Search (RQ-Algorithm)

Each range query is determined by 2 tuples \( q_a \) and \( q_e \) which (visually) specify the left upper and right lower edge of the query region.

1. Begin with \( q_a \) and compute the respective Z-Region.
2. Load the respective page and apply the query predicate to all tuples within the page.
3. Compute the next region of the Z-Kurzve, that lies within the query region.
4. Repeat 2. and 3. until the end address of the edited Z-Region is bigger than \( q_e \) (and also contains the end point of the query region).
UB-Tree: Region Search

- Step 3. (Computation of the intersection points of the Z-Curve with the query region)
  - Appears critical at first glance;
  - In fact efficiently solved by using "a few" bit operations (and without disk accesses) in linear time (linear in the length of the Z-Values)
UB-Tree: Visual Region Search
UB-Tree and MHC

Extensions of the UB-Tree for Data Warehouses:

- Multidimensional Hierarchical Clustering (MHC)
- Supports hierarchically organized dimensions, so that all advantages of the UB-Tree remain
MHC: Principle

- Total order for hierarchy
- Assignment of a unique number to each leaf element of the hierarchy
- Elements of the same subtree contain grouped numbers (Clustering)

Computation:
- Each element of a hierarchy level contains a number (surrogate)
- For leaf element: linking the surrogates together (binary representation) $\rightarrow$ multicomponent surrogate
MHC Example

Top

Wein (1)

Rotwein (1) Weißwein (2) Rose (3)

Bordeaux (1) Burgunder (2) Rivaner (1)

111 112 121 ...

Bier (2)

Pils (1) Alt (2)
MHC Usage

- Multicomponent surrogate as
  - Key for tuples in the fact table
  - Index attribute for UB-Tree

- Example: Range query
  - Minimum und maximum multicomponent surrogate as interval for restriction
Summary

- Index structures allow improved multidimensional queries
- One dimensional index structures do not suffice
- B-Tree, Hash approaches and extensions are one dimensional
- Tree- and Hash approaches can be adjusted for multidimensionality
  - kdB-Tree, UB-Tree, R-Tree
  - Grid Files, MDH